

# Lasing without Inversion in Circuit Quantum Electrodynamics

M. Marthaler<sup>1</sup>, Y. Utsumi<sup>2</sup>, D. S. Golubev<sup>3</sup>, A. Shnirman<sup>4,5</sup>, and Gerd Schön<sup>1,3,5</sup>

<sup>1</sup>*Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

<sup>2</sup>*Department of Physics Engineering, Faculty of Engineering, Mie University, Tsu, Mi-e, 514-8507, Japan*

<sup>3</sup>*Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*

<sup>4</sup>*Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

<sup>5</sup>*DFG-Center for Functional Nanostructures (CFN), Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

We study the photon generation in a transmission line oscillator coupled to a driven qubit in the presence of a dissipative electromagnetic environment. It has been demonstrated previously that a population inversion in the qubit may lead to a lasing state of the oscillator. Here we show that the circuit can also exhibit the effect of “lasing without inversion”. This is possible since the coupling to the dissipative environment enhances photon emission as compared to absorption, similar to the recoil effect which was predicted for atomic systems. While the recoil effect is very weak, and so far elusive, the effect described here should be observable with present circuits. We analyze the requirements for the system parameters and environment.

**Introduction.** The basic “circuit quantum electrodynamics” (cQED) system consists of a superconducting qubit coupled to a transmission line oscillator. The former replaces the atom, the latter the radiation field of the traditional quantum electrodynamics setup<sup>1</sup>. In recent experiments<sup>2</sup> a cQED version of the single-atom maser was realized. For that purpose a superconducting single-electron transistor is coupled to a transmission-line resonator with frequencies in the microwave regime. In this particular case a population inversion of the two charge states, which are in resonance with the oscillator, is achieved by pumping via a third state<sup>3–7</sup>. The same setup can also be used to create non-classical photon states in the resonator<sup>8</sup> or, when coupled to a mechanical oscillator, it is one of the prime candidates to create and observe non-classical states in macroscopic objects<sup>9,10</sup>.

The exchange of energy quanta between the atom and the cavity, or in the present case qubit and resonator, including the escape of photons from the cavity, is governed by the balance

$$\frac{P_{n+1}}{P_n} = \frac{\Gamma_{\text{ph}}^+ P_{\uparrow} + \kappa \bar{n}}{\Gamma_{\text{ph}}^- P_{\downarrow} + \kappa (\bar{n} + 1)}. \quad (1)$$

Here  $P_n$  is the probability to find  $n$  photons in the cavity, while  $P_{\uparrow/\downarrow}$  are the occupation probabilities of the atomic levels. The rates of stimulated photon emission and absorption are given by  $\Gamma_{\text{ph}}^+$  and  $\Gamma_{\text{ph}}^-$ , respectively, while  $\kappa$  is the decay rate of photons in the oscillator with  $\bar{n}$  thermal photons. In thermal equilibrium the occupation probability decreases as a function of the photon number,  $P_{n+1}/P_n = \bar{n}/(\bar{n} + 1)$ . In contrast, the lasing state is characterized by a sharp peak of  $P_n$  at nonzero values of  $n$ , and hence  $P_{n+1} > P_n$  below the peak. In the optical domain the absorption and emission rates are roughly equal. Hence the condition that  $P_n$  should grow with  $n$  can only be achieved if a population inversion,  $P_{\uparrow} > P_{\downarrow}$ , is created. This is done in a pump process which typically involves a third level. Examples are the early microwave lasers based on ammonia molecules<sup>11</sup> as well as the recent

cQED lasers with charge qubits, where the pumping is achieved by quasiparticle tunneling processes via a third state with an extra quasiparticle<sup>2</sup>.

As lasing technology progressed it became clear that population inversion is not really needed. One of the earliest schemes of lasing without inversion (LWI) is the pumping of Rabi sidebands<sup>12,13</sup>; its cQED analog was demonstrated and discussed in Refs. 14 and 15. However, this process is not strictly LWI, since closer inspection reveals that it is based on a population inversion in the dressed states basis<sup>16</sup>. LWI without hidden inversion can only be achieved by breaking the symmetry of photon emission and absorption<sup>17</sup>. The only experimental demonstrations of this symmetry breaking were accomplished by using several external fields, which change the absorption and emission profile of the bare atom<sup>18,19</sup>.

In this paper we will employ a fundamentally different method that is more similar to the idea of using the shift of the emission peak caused by the recoil effect on the atom<sup>20</sup>. In this case the atom always absorbs energy for both emission and absorption. This effect has never been verified experimentally in conventional lasing setups, since it requires large frequencies and/or low temperatures such that the recoil energy exceeds the temperature.<sup>21</sup> With cQED systems the situation appears more promising due to two differences to quantum optics. One reason is that there are many intrinsic sources of non-classical noise suppressing energy emission as compared to absorption. The other is the strong coupling between the qubit and the resonator, which makes it possible to create a lasing situation even in the presence of strong noise.

In the following we will introduce the system and the methods needed to describe the shift of the emission peak. Our methods reproduce the standard lasing results<sup>22</sup> but, in addition, describe the effect of strong noise as long as the photon number is not too large. We will then demonstrate that it is possible to use a noisy qubit to create a non-thermal photon state in the oscillator and that the oscillator field exhibits

the linewidth narrowing characteristic for the lasing state. We describe the requirements for the system and environment needed to observe inversionless lasing. Finally, we will describe a specific experimental setup based on a superconducting circuit.

**The System:** The basic mechanism which leads to lasing without inversion is the shift of the photon emission line due to the coupling to a dissipative environment. This effect may play a role in every setup that can be described by the extension of the Jaynes–Cummings Hamiltonian ( $\hbar = 1$ ),

$$H = \omega_0 a^\dagger a + g(a^\dagger \sigma_- + a \sigma_+) + \frac{1}{2} \Delta E \sigma_z + \frac{1}{2} X(t) \sigma_z. \quad (2)$$

Here  $a$  is the annihilation operator for the oscillator mode with frequency  $\omega_0$ , the Pauli matrices  $\sigma_i$  act on the states  $|\uparrow\rangle$  and  $|\downarrow\rangle$ , the qubit energy splitting and the strength of the coupling between oscillator and qubit are denoted by  $\Delta E$  and  $g$ , respectively. We also account for fluctuations  $X(t)$  of the energy splitting of the qubit. They may originate from the electrodynamic environment and, together with strong coupling, make inversionless lasing possible. The noise  $X(t)$  is characterized by its correlator,

$$\langle X(t)X(0) \rangle = \frac{1}{\pi} \int d\omega J(\omega) \left[ \coth\left(\frac{\omega}{2k_B T}\right) \cos \omega t - i \sin \omega t \right], \quad (3)$$

where  $T$  is the base temperature. To be specific we assume that the fluctuations result from an Ohmic environment; taking into account a capacitive coupling to the system it leads to a spectral function of the form

$$J(\omega) = \epsilon_C \omega \times \frac{\omega_R}{\omega^2 + \omega_R^2}, \quad (4)$$

parametrized by a coupling strength  $\epsilon_C$  and cut-off frequency  $\omega_R$ .

Here we consider situations where the noise is strong. Therefore we proceed using the polaron transformation  $U = \exp[-i\sigma_z \int_{-\infty}^t X(t') dt']$ , which yields

$$\begin{aligned} H &= H_0 + H_g, \\ H_0 &= \omega_0 a^\dagger a + \frac{1}{2} \Delta E \sigma_z, \\ H_g &= g \left[ \sigma_+ a e^{-i \int_{-\infty}^t X(t') dt'} + \text{h.c.} \right]. \end{aligned} \quad (5)$$

In the following we will assume that the Hamiltonian part  $H_g$  leads to incoherent transitions, which we analyze in the spirit of the so-called  $P(E)$ –Theory<sup>23</sup> by an appropriate Liouville term in the Master equation. This somewhat unusual treatment of the coupling between oscillator and qubit differs from the approaches to lasing followed, e.g., by Refs. 6, 14, 22, and 24 where the coupling is treated as part of the coherent time evolution. However, as we will demonstrate below our master equation approach is valid in a broad parameter regime,

and it reproduces results for the average photon number and phase coherence time which are typical for lasing.

**Master Equation.** Our system is described by the Master equation

$$\dot{\rho} = i[H_0, \rho] + [\mathcal{L}_{\text{diss}} + \mathcal{L}_g + \mathcal{L}_{\text{pump}}] \rho. \quad (6)$$

It contains three Liouville operators, corresponding to an incoherent pump process,  $\mathcal{L}_{\text{pump}}\rho$ , photon emission and absorption,  $\mathcal{L}_g\rho$ , and dissipation in the oscillator,  $\mathcal{L}_{\text{diss}}\rho$ . Incoherent pumping is described by

$$\begin{aligned} \mathcal{L}_{\text{pump}}\rho &= \Gamma_{\text{up}}(2\sigma_+\rho\sigma_- - [\sigma_-\sigma_+, \rho]_+) \\ &\quad + \Gamma_{\text{down}}(2\sigma_-\rho\sigma_+ - [\sigma_+\sigma_-, \rho]_+), \end{aligned} \quad (7)$$

where  $[\cdot]_+$  denotes an anti-commutator. Without coupling to the oscillator,  $g = 0$ , the occupation probabilities of the atom,  $P_\uparrow = \langle \uparrow | \rho | \uparrow \rangle$  and similar  $P_\downarrow$ , would be given by  $P_\uparrow/P_\downarrow = \Gamma_{\text{up}}/\Gamma_{\text{down}}$ . That means, as long as  $\Gamma_{\text{down}} \geq \Gamma_{\text{up}}$  the pumping does not produce a population inversion. For later use we introduce the parameter  $\Gamma_{\text{pump}} = \Gamma_{\text{up}} + \Gamma_{\text{down}}$  and define the population inversion coefficient  $D_0 = (\Gamma_{\text{up}} - \Gamma_{\text{down}})/\Gamma_{\text{pump}}$ .

The crucial effect to be considered here is the emission and absorption of photons, i.e., the transitions between the states  $|\uparrow\rangle|n\rangle$  and  $|\downarrow\rangle|n+1\rangle$  induced by the Hamiltonian  $H_g$ . To calculate the rates we expand the time evolution of the density matrix up to second order in  $g$ , and consider the level broadening caused by the pump. In this case the Liouvillian is given by

$$\begin{aligned} \mathcal{L}_g\rho &= \frac{\Gamma_{\text{ph}}^+}{2} (2\sigma_- a^\dagger \rho a \sigma_+ - [a \sigma_+ \sigma_- a^\dagger, \rho]_+) \\ &\quad + \frac{\Gamma_{\text{ph}}^-}{2} (2a \sigma_+ \rho \sigma_- a^\dagger - [\sigma_- a^\dagger a \sigma_+, \rho]_+), \end{aligned} \quad (8)$$

with photon emission and absorption rates  $\Gamma_{\text{ph}}^\pm = g^2 S_{\text{ph}}(\pm\delta\omega)$ . They depend on the spectral function

$$\begin{aligned} S_{\text{ph}}(\omega) &= \int_{-\infty}^{\infty} C_{\text{ph}}(t) e^{-\Gamma_{\text{pump}}|t|/2} e^{i\omega t} dt, \\ C_{\text{ph}}(t) &= \exp\left\{ -\frac{1}{\pi} \int d\omega \frac{J(\omega)}{\omega^2} \right. \\ &\quad \times \left. \left[ 2 \sin^2\left(\frac{\omega t}{2}\right) \coth\left(\frac{\omega}{2k_B T}\right) + i \sin \omega t \right] \right\}, \end{aligned} \quad (9)$$

at the frequency given by the detuning  $\delta\omega = \Delta E - \omega_0$ . We assume that a fairly strong pump is applied to the qubit, as it was the case in the experiments of Ref. 2. In this case the lowest order expansion in  $H_g$  converges for all combinations of  $\epsilon_C$  and  $\omega_R$ , as long as  $g\sqrt{n} \ll \Gamma_{\text{pump}}$ . Including the level broadening caused by the pump is crucial to reproduce the standard lasing results in the limit of weak noise  $\epsilon_C \rightarrow 0$ ,  $\omega_R \rightarrow \infty$ . One should note that even for  $g\sqrt{n} > \Gamma_{\text{pump}}$  our master equation is still valid for  $\epsilon_C = 0$  in the stationary limit and for  $g^2 n \ll \sqrt{k_B T} \epsilon_C \omega_R$  at all times.

Finally, dissipation in the oscillator is described by the standard Lindblad operator<sup>24</sup>,

$$\mathcal{L}_{\text{diss}}\rho = \frac{\kappa}{2}(\bar{n} + 1)(2a\rho a^\dagger - [a^\dagger a, \rho]_+) \quad (10)$$

$$+ \frac{\kappa}{2}\bar{n}(2a^\dagger\rho a - [aa^\dagger, \rho]_+),$$

with dissipation strength determined by the decay rate  $\kappa$  and the temperature entering the thermal number of photons  $\bar{n} = (\exp(\omega_0/k_B T) - 1)^{-1}$ .

The master equation (6) can be solved by numerically diagonalizing it in the eigenbasis of  $H_0$ . On the other hand, there are two standard methods to solve the Master equation analytically<sup>22</sup>. One method is to consider the equation of motion of operator averages and the second method is to adiabatically eliminate qubit states and derive an effective equation of motion for the oscillator photon number states,  $\rho_{nn'} = \langle n | \text{Tr}_{\text{qubit}} \rho | n' \rangle$ , where the trace is taken over the qubit eigenstates  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . With the combination of these two methods we can derive analytical results for all quantities we are interested in. To find a closed set of equations for the operator averages we have to make the semi-classical approximation  $\langle a^\dagger a \sigma_z \rangle \approx \langle a^\dagger a \rangle \langle \sigma_z \rangle$ . This is a good approximation in the stationary limit and allows us to derive an analytical expression for the average photon number in the system  $\langle n \rangle = \langle a^\dagger a \rangle$ . To describe the time evolution of the system we use the adiabatic elimination of the qubit degrees of freedom. This method accurately describes the time-evolution of the off-diagonal matrix elements even for  $\Gamma_{\text{ph}}^+ \neq \Gamma_{\text{ph}}^-$ .

**Lasing Without Inversion.** To demonstrate that inversionless lasing is possible in cQED we show the average photon number in the oscillator for three illustrative cases (see fig. 1). We always choose the detuning between oscillator and qubit such that the photon emission rate  $\Gamma_{\text{ph}}^+$  is maximized. The cases that are shown correspond to the standard lasing situation with a noiseless qubit,  $\epsilon_C \rightarrow 0, \omega_R \rightarrow \infty$ , and to situations with strong noise. Without noise the emission and absorption rates are equal and we can only achieve heating of the oscillator. For strong noise we tune the system such that there is additional energy available if a photon is created,  $\delta\omega = \Delta E - \omega > 0$ . The additional energy is given to the environment. This means that for the qubit to absorb a photon the environment has to deliver the energy  $\delta\omega$ . Since we consider an environment at low temperatures, we get strong suppression of photon absorption and hence there is a strong imbalance between the emission and the absorption rate,  $\Gamma_{\text{ph}}^+ \gg \Gamma_{\text{ph}}^-$ . So despite the fact that no population inversion is created in the qubit a large number of photons is excited in the oscillator. For the noiseless case the number of photons only starts to rise strongly as we achieve inversion,  $D_0 > 0$ .

The optimal condition for inversionless lasing is reached if the coupling to noise is stronger than the level broadening caused by the pump  $\epsilon_C \gg \Gamma_{\text{pump}}$  and we maximize the asymmetry due to small temperatures by

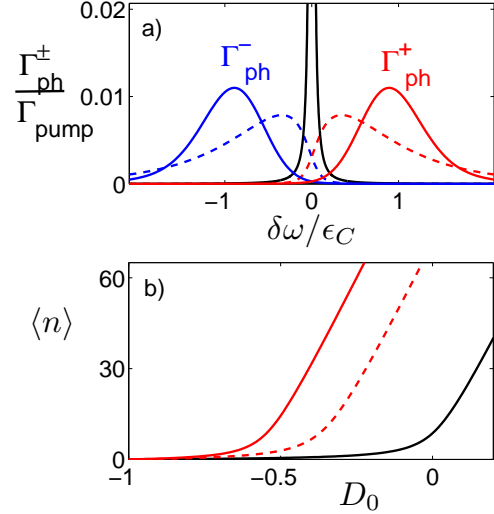


FIG. 1. a) The photon emission and absorption rate,  $\Gamma_{\text{ph}}^+$  and  $\Gamma_{\text{ph}}^-$ , as a function of the frequency detuning  $\delta\omega$ . Without noise,  $\omega_R \rightarrow \infty$ , the two rates are equal (black line). Dashed, red and blue lines show the emission and absorption rate, respectively, for  $\Gamma_{\text{pump}}/\epsilon_C = 0.0325$  and  $\omega_R/\epsilon_C = 0.5$ . Full, red and blue lines show the rates for  $\Gamma_{\text{pump}}/\epsilon_C = 0.0325$  and  $\omega_R/\epsilon_C = 0.05$ . b) The average photon number  $\langle n \rangle$  as function of the inversion coefficient  $D_0$ . The black line is the result for a qubit without noise,  $\omega_R \rightarrow \infty$ , at  $\delta\omega = 0$ . The red lines are the results under the influence of noise, for  $\omega_R = 0.5$  at  $\delta\omega/\epsilon_C = 0.05$  (dashed line) and for  $\omega_R = 0.05$  at  $\delta\omega/\epsilon_C = 1$  (full line). The frequency detuning has been chosen to maximize the photon number. We used the parameters  $\Gamma_{\text{pump}}/\epsilon_C = 0.0325$ ,  $k_B T/\epsilon_C = 0.05$ ,  $g/\epsilon_C = 0.01$ ,  $\kappa/\epsilon_C = 8.125 \times 10^{-5}$ .

choosing  $\epsilon_C \gg k_B T \gtrsim \omega_R$ . For each photon that is emitted or absorbed the energy  $\epsilon_C$ <sup>23</sup> is given to the environment. Therefore the maximum for photon emission is at  $\delta\omega = \epsilon_C$  where photon absorption is strongly suppressed. The rates (9) obey detailed balance only for the system in equilibrium,  $\Gamma_{\text{pump}} = 0$ . However, if a pump is applied the extra broadening has to be taken into account. At the maximum of the photon emission rate we can approximate the rates by

$$\Gamma_{\text{ph}}^+ \approx \frac{g^2}{\sqrt{k_B T \epsilon_C}}, \quad \Gamma_{\text{ph}}^- \approx \frac{g^2 \Gamma_{\text{pump}}}{\epsilon_C^2}. \quad (11)$$

From these rates we can formulate the threshold condition defined in eq. (1) for  $\bar{n} \ll 1$  as

$$\frac{P_{n+1}}{P_n} = \frac{1}{\sqrt{k_B T} \Gamma_{\text{pump}}} \frac{g^2 \epsilon_C^{3/2} \Gamma_{\text{up}}}{g^2 \Gamma_{\text{down}} + \kappa \epsilon_C^2}. \quad (12)$$

Using the explicit form of the photon emission and absorption rate (11) we will now summarize the two crucial prerequisites for inversionless lasing: strong noise at small temperatures and strong coupling. The first condition is needed to create an imbalance between photon

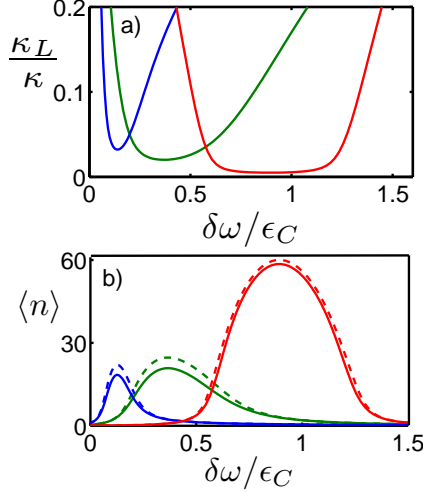


FIG. 2. a) The phase correlation decay rate  $\kappa_L$  as a function of the frequency detuning  $\delta\omega$ . The values of the cut-off frequency are  $\omega_R/\epsilon_C = 0.05$  (red),  $\omega_R/\epsilon_C = 0.5$  (green),  $\omega_R/\epsilon_C = 2$  (blue). b) The average photon number  $\langle n \rangle$  as a function of the frequency detuning  $\delta\omega$  with color coding as in a). Full lines display the results of a numerical solution of the master equation, dashed lines are the approximate analytical solution (15). The system parameters are  $\Gamma_{\text{pump}}/\epsilon_C = 0.0325$ ,  $k_B T/\epsilon_C = 0.05$ ,  $g/\epsilon_C = 0.01$ ,  $\kappa/\epsilon_C = 8.125 \times 10^{-5}$ ,  $D_0 = -0.25$ .

emission and absorption. Comparing the absorption and emission rate given by eq. (11) we find

$$\frac{\epsilon_C^{3/2}}{k_B T} \gg \Gamma_{\text{pump}}. \quad (13)$$

The second condition, strong coupling, can be formulated by comparing photon emission with the oscillator decay rate. This yields

$$\frac{g^2}{\sqrt{\epsilon_C k_B T}} \gg \kappa. \quad (14)$$

One should note that the condition of strong coupling requires only a comparison of coupling to noise strength. We still want the system to be within the limit of the rotating wave approximation,  $g \ll \omega$ , in contrast to the so called ultra-strong coupling regime recently observed in Ref. 25.

For the conditions of strong noise (13) and strong coupling (14) we additionally assumed  $\sqrt{k_B T \epsilon_C} \gg \omega_R$ . However this is not strictly necessary as is shown in fig. 2. Here we show the average photon number obtained from a numerical calculation of the photon emission and absorption rates. Even for larger cut-off frequencies we can create a lasing state in the oscillator. However, the photon number gets maximal for a small cut-off frequency.

For unequal photon creation and annihilation rates and assuming that all conditions formulated in eqns. (12),

(13) and (14) apply we find the average photon number to be

$$\langle n \rangle \approx \frac{\Gamma_{\text{up}} \Gamma_{\text{ph}}^+ - \Gamma_{\text{down}} \Gamma_{\text{ph}}^-}{(\Gamma_{\text{ph}}^+ + \Gamma_{\text{ph}}^-) \kappa} + \mathcal{O}(\kappa^0). \quad (15)$$

This results confirms our previous discussion. To get a high photon number we need a low decay rate  $\kappa$  in the oscillator and  $\Gamma_{\text{up}} \Gamma_{\text{ph}}^+ - \Gamma_{\text{down}} \Gamma_{\text{ph}}^- > 0$ . We also see the standard lasing condition emerge. For equal photon creation and annihilation rates,  $\Gamma_{\text{ph}}^+ = \Gamma_{\text{ph}}^-$ , we would need  $\Gamma_{\text{up}} > \Gamma_{\text{down}}$  to get a large photon number. If all orders of  $\kappa$  are taken into account our result for the average photon number exactly reproduces the standard lasing results in the limit  $\epsilon_C \rightarrow 0, \omega_R \rightarrow \infty$ .

One of the characteristic properties of a laser is the phase coherence, which is expressed in the small decay rate of the phase correlator  $C(t) = \langle a(t)a(0) \rangle$ . In the Markovian limit the correlator decays exponentially with the rate  $\kappa_L$ ,  $C(t) \propto e^{-\kappa_L t}$ . To find the correlator we use the regression theorem

$$\langle a^\dagger(t)a(0) \rangle = \text{Tr} [a^\dagger \rho_a(t)], \quad (16)$$

where  $\rho_a(t)$  is a density matrix which follows the time evolution given by the master equation (6) with the initial condition  $\rho_a(0) = a \rho_{\text{st}}$ . After adiabatically removing the time-evolution of the qubit, the problem is reduced to the decay of the matrix elements  $\rho_{nn+1}$ . For large photon number the decay rate is given by

$$\kappa_L = \frac{1}{\langle n \rangle} \frac{(\kappa \Gamma_{\text{pump}} + \Gamma_{\text{down}} \Gamma_{\text{ph}}^- + \Gamma_{\text{up}} \Gamma_{\text{ph}}^+)}{8 \Gamma_{\text{pump}}}. \quad (17)$$

Even for  $\Gamma_{\text{ph}}^+ \neq \Gamma_{\text{ph}}^-$  the decay rate is inversely proportional to the photon number as it is the standard case for a laser. In the limit of weak noise,  $\epsilon_C \rightarrow 0, \omega_R \rightarrow \infty$ , this result reproduces the standard lasing results, as long as we stay within the general limit of validity of our master equation. For a discussion of the phase correlation decay rate in the limit of strong coupling and weak noise, see e.g., Ref. 7.

In fig. 2 we show the phase correlation decay rate  $\kappa_L$  and the corresponding average photon number  $\langle n \rangle$  as a function of the frequency detuning  $\delta\omega$ . The numerical and analytical results for the average photon number are in good agreement; for the decay rate the comparison is not shown but there is also qualitative agreement. Results are shown for different values of the cut-off frequency  $\omega_R$ , and one can see that the enhancement of photon number is the stronger, the smaller is  $\omega_R$ . On the other hand, even for larger values of  $\omega_R$  lasing is possible with a strongly reduced phase correlation decay rate.

**Outlook.** A specific system where the conditions for lasing without inversion can be realized is the single artificial-atom laser investigated by Astafiev *et al.*<sup>2</sup>. This system consists of a charge qubit coupled to an oscillator. An applied transport voltage induces quasiparticle

tunneling via a third state which can be used to create a population inversion in the qubit, which has led to the observed lasing. With the same setup it would also be possible to only create an enhanced population of the excited qubit level without inversion. The noise spectrum needed for inversionless lasing can be created by coupling the charge qubit to an external resistor (see e.g. Ref. 26). To reach a low cut-off frequency it is necessary to use a resistor with a large resistance, as it has been demon-

strated, e.g., in Ref. 27. The strong coupling needed to satisfy the requirement (14) is standard in most cQED experiments<sup>1,2,25</sup>.

We gratefully acknowledge discussions with V. Brosco and Y. Nakamura. Y. Utsumi acknowledges supported by the Strategic International Cooperative Program of the Japan Science and Technology Agency (JST) and by the German Science Foundation (DFG).

- 
- <sup>1</sup> A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf, *Nature* **431**, 162 (2004); J. M. Fink, M. Göppl, M. Baur, R. Bianchetti, P. J. Leek, A. Blais and A. Wallraff, *Nature* **454**, 315 (2008); M. Hofheinz, H. Wang, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, J. Wenner, J. M. Martinis and A. N. Cleland, *Nature* **459**, 546 (2009).
  - <sup>2</sup> O. Astafiev, K. Inomata, A.O. Niskanen, T. Yamamoto, Yu. A. Pashkin, Y. Nakamura and J. S. Tsai, *Nature* **449**, 588 (2007).
  - <sup>3</sup> D. A. Rodrigues, J. Imbers and A. D. Armour, *Phys. Rev. Lett.* **98**, 067204 (2007).
  - <sup>4</sup> M. P. Blencowe, J. Imbers and A. D. Armour, *New J. Phys.* **7**, 236 (2005); D. A. Rodrigues and A. D. Armour, *New J. Phys.* **7**, 251 (2005).
  - <sup>5</sup> A. A. Clerk and S. Bennet, *New J. Phys.* **7**, 238 (2005).
  - <sup>6</sup> S. André, V. Brosco, M. Marthaler, A. Shnirman and G. Schön, *Phys. Script.* **T 137**, 014016 (2009).
  - <sup>7</sup> S. André, P.-Q. Jin, V. Brosco, J. H. Cole, A. Romito, A. Shnirman and G. Schön, *Phys. Rev. A* **82**, 053802 (2010).
  - <sup>8</sup> M. Marthaler, G. Schön and A. Shnirman, *Phys. Rev. Lett.* **101**, 147001 (2008).
  - <sup>9</sup> M. D. LaHaye, J. Suh, P. M. Echternach, K. C. Schwab and M. L. Roukes, *Nature* **459**, 960 (2009).
  - <sup>10</sup> M. Blencowe, *Phys. Rep.* **395**, 159 (2004).
  - <sup>11</sup> Charles H. Townes, J. P. Gordon and H. J. Zeiger, *Phys. Rev.* **95**, 282 (1954).
  - <sup>12</sup> S. G. Rautian and I. I. Sobel'man, *Zh. eksp. teor. Fiz.* **41**, 328 (1961).
  - <sup>13</sup> F. Y. Wu, S. Ezekiel, M. Ducloy and B. R. Mollow, *Phys. Rev. Lett.* **38**, 1077 (1977).
  - <sup>14</sup> J. Hauss, A. Fedorov, C. Hutter, A. Shnirman and G. Schön, *Phys. Rev. Lett.* **100**, 037003 (2008).
  - <sup>15</sup> M. Grajcar, S. H. W. Van der Ploeg, A. Izmalkov, E. IL'Ichev, H.-G. Meyer, A. Fedorov, A. Shnirman and G. Schön, *Nat. Phys.* **4**, 612 (2008).
  - <sup>16</sup> J. Mompert and R. Corbalan, *J. Opt. B* **2**, R7 (2000).
  - <sup>17</sup> O. Kocharovska and Ya. I. Khanin, *JETP Lett.* **48**, 630 (1988).
  - <sup>18</sup> A. Nottelman, C. Peters and W. Lange, *Phys. Rev. Lett.* **70**, 1783 (1993).
  - <sup>19</sup> A. S. Zibrov, M. D. Lukin, D. E. Nikonov, L. Hollberg, M. O. Scully, V. L. Velichansky and H. G. Robinson, *Phys. Rev. Lett.* **62**, 2813 (1995).
  - <sup>20</sup> D. Marcuse, *Proc. IEEE* **51**, 849 (1963).
  - <sup>21</sup> Such a requirement is sometimes considered as inversion of the environment, see e.g. O. Kacharovskaya, *Phys. Rep.* **219**, 175 (1992).
  - <sup>22</sup> "Quantum Optics", M. O. Scully and M. S. Zubairy, Cambridge University Press (1997).
  - <sup>23</sup> G.-L. Ingold and Y. V. Nazarov, in *Single Charge Tunneling*, eds. H. Grabert and M. H. Devoret, Plenum Press, New York (1992).
  - <sup>24</sup> "Statistical Methods in Quantum Optics", H. J. Carmichael, Springer (2001).
  - <sup>25</sup> T. Niemczyk, F. Deppe, H. Huebl, E. P. Menzel, F. Hocke, M. J. Schwarz, J. J. Garcia-Ripoll, D. Zueco, T. Hämmer, E. Solano, A. Marx and R. Gross, *Nature Physics* **6**, 772 (2010).
  - <sup>26</sup> G.-L. Ingold, P. Wyrowski and H. Grabert, *Z. Phys. B* **85**, 443 (1991).
  - <sup>27</sup> J. Delahaye, J. Hassel, R. Lindell, M. Sillanpää, M. Paalanen, H. Seppä and P. Hakonen, *Science* **299**, 1045 (2003).